Exercise 7

Verify the given linear approximation at $a = 0$. Then determine the values of x for which the linear approximation is accurate to within 0.1.

$$\ln(1 + x) \approx x$$

Solution

Plugging in $x = 0$ to the function yields $\ln(1 + 0) = 0$, so $(0, 0)$ is the point on the curve that the tangent line goes through. Taking the derivative of the function yields

$$\frac{d}{dx}\ln(1 + x) = \frac{1}{1 + x} \cdot \frac{d}{dx}(1 + x) = \frac{1}{1 + x} \cdot 1 = \frac{1}{1 + x}.$$

Set $x = 0$ to get the slope of the tangent line.

$$\frac{d}{dx}\ln(1 + x)\bigg|_{x=0} = \frac{1}{1 + 0} = 1$$

Use the point-slope formula to get the equation of this line.

$$y - 0 = 1(x - 0)$$

$$y = x$$

As a result, the linearization to $\ln(1 + x)$ at 0 is

$$L(x) = x.$$

Now find the values of x for which the linear approximation is accurate to within 0.1.

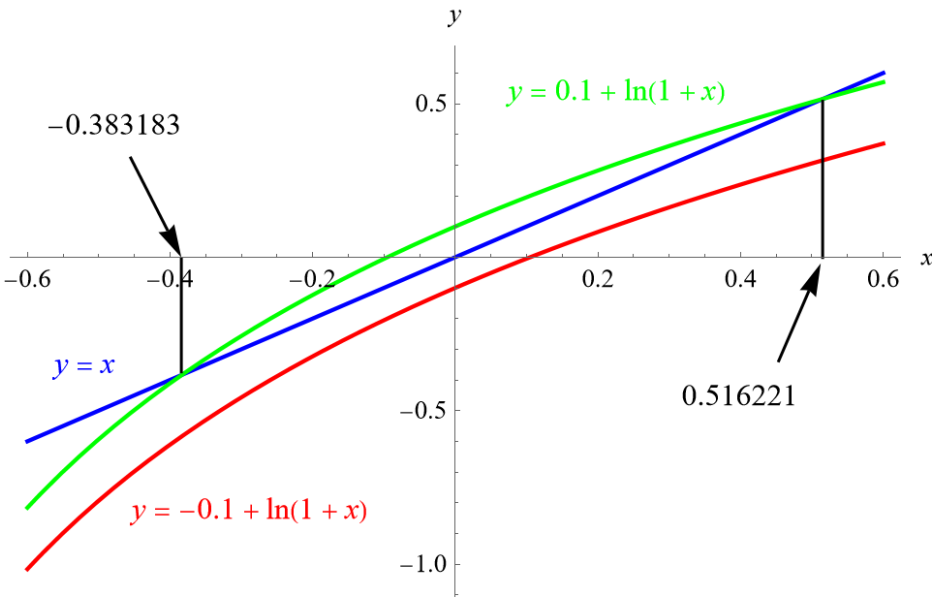
$$|\ln(1 + x) - x| < 0.1$$

$$|x - \ln(1 + x)| < 0.1$$

$$-0.1 < x - \ln(1 + x) < 0.1$$

$$-0.1 + \ln(1 + x) < x < 0.1 + \ln(1 + x)$$

Plot each of these functions versus x.



The linear approximation stays between the curves for

$$-0.383183 < x < 0.516221.$$

This is the interval that the linear approximation is accurate to within 0.1.